A BAYESIAN MODEL FOR DETERMINING THE OPTIMAL TEST STRESS FOR A SINGLE TEST UNIT

by

H. F. Martz, Jr. Texas Tech University

and

M. S. Waterman, Los Alamos Scientific Laboratory

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H. F. Martz, Jr.
Principal Investigator

Department of Industrial Engineering
Texas Tech University
Lubbock, Texas 79409

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ABSTRACT

Consider the case of a single test unit which must be tested at some level of test stress. Suppose that the test stress level is free to be determined, and that only the survival or nonsurvival of the unit is observed. It is assumed that the unit is designed to withstand a known and specified design stress level. A Bayesian model is developed for determining the required level of test stress which maximizes the expected probability of survival at the design stress level. Engineering experience from similar past tests on similar units is used to fit the model. A practical application illustrates the method. The sensitivity of the procedure to changes in the parameters used in fitting the model is also examined. The procedure is fairly insensitive to three parameters required in fitting the model in the example.

I. INTRODUCTION

In many engineering testing situations, only a single unit is available for testing. The unit may be a component, subsystem or complete system. Suppose that a single test unit is to be tested at some level of a single test stress which is free to be selected by the test engineer. Further, suppose that the unit is designed to withstand a known and specified level of design stress. It is further assumed that, once the test is conducted, only the survival or nonsurvival of the unit is observed.

For example, consider the case of a fuel container of a radioisotope thermoelectric generator (RTG) system, which is the power supply for a space satellite. The radioisotope fuel container is designed to withstand a certain impact onto flat plate steel, such as might occur during a launch pad overpressure accident. As part of the required safety analyses, tests designed to simulate such an accident must be performed. A prototypic unit, using simulated fuel, is impacted onto flat plate steel at some test velocity which must be determined. This test velocity may or may not be taken to be the design velocity. According to a precise definition of failure, e.g., if the unit ruptures to the extent that one or more fuel elements are exposed, the unit either survives or fails the test. This example will be further considered in Section IV.

The model to be developed incorporates the following aspects. Suppose that the test unit survives a test stress which exceeds the design stress. It is reasonable that this should increase the experimenter's confidence in the ability of similar units to survive the design stress. On the other hand, if the test unit is tested at too great a stress, the unit is likely to fail, thus providing little information about the unit's ability to survive the design stress. The model effectively trades between these two alternatives in seeking the optimum desired level of test stress The precise definition of "optimum" will be discussed in the next section

The philosophy of the proposed model is to test at a high enough stress level to provide assurance but not failures. This contrasts with the usual statistical philosophy which is to test at various levels, some of which are high enough to insure failures. Of course, more than one unit must be tested in this case. Easterling (1975) develops an over-test procedure, referred to as a "sensitivity test", which is based on such a statistical philosophy. Much of the literature on accelerated life testing considers the effect of stress on certain failure characteristics. An excellent bibliography on accelerated life testing is provided by Lowe and Waller (1975).

II. THE MODEL

Let S_k denote the event that the test unit survives a test of stress $k \cdot S_o$, where S_o is the given design stress. Also let $P_k = \mathrm{Prob}(S_k)$. A Bayesian approach is used, in which the uncertainty in P_k is expressed by assuming that P_k is a random variable having a modified negative-log gamma prior distribution with probability density function (pdf) given by

$$f(p;\alpha,\beta,\delta) = \frac{p^{(1/\beta k^{\delta})-1}(-\ell np)^{\alpha-1}}{\beta^{\alpha}k^{\alpha\delta}\Gamma(\alpha)}, \quad \begin{array}{c} 0\leq p\leq 1, \\ 0\leq k\leq\infty, \\ 0<\alpha,\beta,\delta \end{array} . \tag{1}$$

This distribution may be derived from the fact that, if λ has a gamma distribution with shape parameter α and scale parameter β, then $\tau\equiv\exp[-\lambda k^{\delta}]$ has the distribution given in (1). Here k is the test stress, expressed in units of design stress S_o. The parameter δ appearing in (1) is used to rescale k, for reasons to be discussed in the next section. The usual negative-log gamma distribution may be obtained by letting $k\equiv\delta\equiv1$. The negative-log gamma distribution has been previously discussed and used in reliability by Springer and Thompson (1965,1967), Mann (1970), and Mastran and Singpurwalla (1974).

The mean and variance of (1) are

$$E(P;\alpha,\beta,\delta) = (1+\beta k^{\delta})^{-\alpha} \tag{2}$$

and

$$V(P;\alpha,\beta,\delta) = (1+2\beta k^{\delta})^{-\alpha} - (1+\beta k^{\delta})^{-2\alpha}, \tag{3}$$

respectively. It might be expected that the mean survival probability curve have a reflected S-shape as a function of k. The mean given in (2) has this property for certain combinations of α,β,δ. It is illustrated in Figure 1 for several choices of α with β and δ computed according to the example in Section IV. Figure 2 shows the standard deviations for the same set of distributions. A procedure for identifying α,β and δ will be presented in the next section.

The distributions of interest are the two posterior distributions of P, conditional on the survival (nonsurvival) of the test unit. By a simple application of Bayes' Theorem, we obtain the two posterior pdf's

$$f(p|S_k;\alpha,\beta,\delta) = \frac{p^{(1/\beta k^{\delta})}(-\ell np)^{\alpha-1}}{\beta^{\alpha}k^{\alpha\delta}\Gamma(\alpha)(1+\beta k^{\delta})^{-\alpha}}, \quad \begin{array}{c} 0<p\leq 1, \\ 0\leq k\leq\infty, \\ \alpha,\beta,\delta>0 \end{array} \tag{4}$$

$$f(p|\tilde{S}_k;\alpha,\beta,\delta) = \frac{(1-p)p^{(1/\beta k^{\delta})-1}(-\ell np)^{\alpha-1}}{\beta^{\alpha}k^{\alpha\delta}\Gamma(\alpha)[1-(1+\beta k^{\delta})^{-\alpha}]}, \qquad \begin{array}{l} 0\le p\le 1, \\ 0\le k\le\infty, \\ \alpha,\beta,\delta>0, \end{array} \tag{5}$$

where $\tilde{S}_k$ denotes the event that the test unit does not survive a test of stress $k \cdot s_0$. The cumulative distribution functions (cdf's) associated with (4) and (5) may be expressed in terms of the chi-square (χ^2) distribution as

$$F(p|S_k;\alpha,\beta,\delta) = \text{Prob}\left\{P\le p|S_k;\alpha,\beta,\delta\right\} = \text{Prob}\left\{\chi^2_{2\alpha} > \frac{-2(1+\beta k^{\delta})\ell np}{\beta k^{\delta}}\right\} \tag{6}$$

and

$$F(p|\tilde{S}_k;\alpha,\beta,\delta) = \text{Prob}\left\{P\le p|\tilde{S}_k;\alpha,\beta,\delta\right\}$$

$$= \frac{\text{Prob}\left\{\chi^2_{2\alpha} > \frac{-2\ell np}{\beta k^{\delta}}\right\} - (1+\beta k^{\delta})^{-\alpha}\text{Prob}\left\{\chi^2_{2\alpha} > \frac{-2(1+\beta k^{\delta})\ell np}{\beta k^{\delta}}\right\}}{1-(1+\beta k^{\delta})^{-\alpha}}, \tag{7}$$

where $\chi^2_{2\alpha}$ denotes a χ^2 random variable with 2α degrees of freedom.

The posterior means of (4) and (5) are easily computed to be

$$E(P_k|S_k;\alpha,\beta,\delta) = \left(\frac{1+2\beta k^{\delta}}{1+\beta k^{\delta}}\right)^{-\alpha} \tag{8}$$

and

$$E(P_k|\tilde{S}_k;\alpha,\beta,\delta) = \frac{(1+\beta k^{\delta})^{-\alpha} - (1+2\beta k^{\delta})^{-\alpha}}{1-(1+\beta k^{\delta})^{-\alpha}}. \tag{9}$$

The unconditional probability of survival and nonsurvival of the test unit, when tested at stress level k, are

$$\text{Prob}(S_k) = (1+\beta k^{\delta})^{-\alpha}, \tag{10}$$

and

$$\text{Prob}(\tilde{S}_k) = 1-(1+\beta k^{\delta})^{-\alpha}, \tag{11}$$

respectively. Note that, since only a single unit is to be tested, (10) is the same as (2).

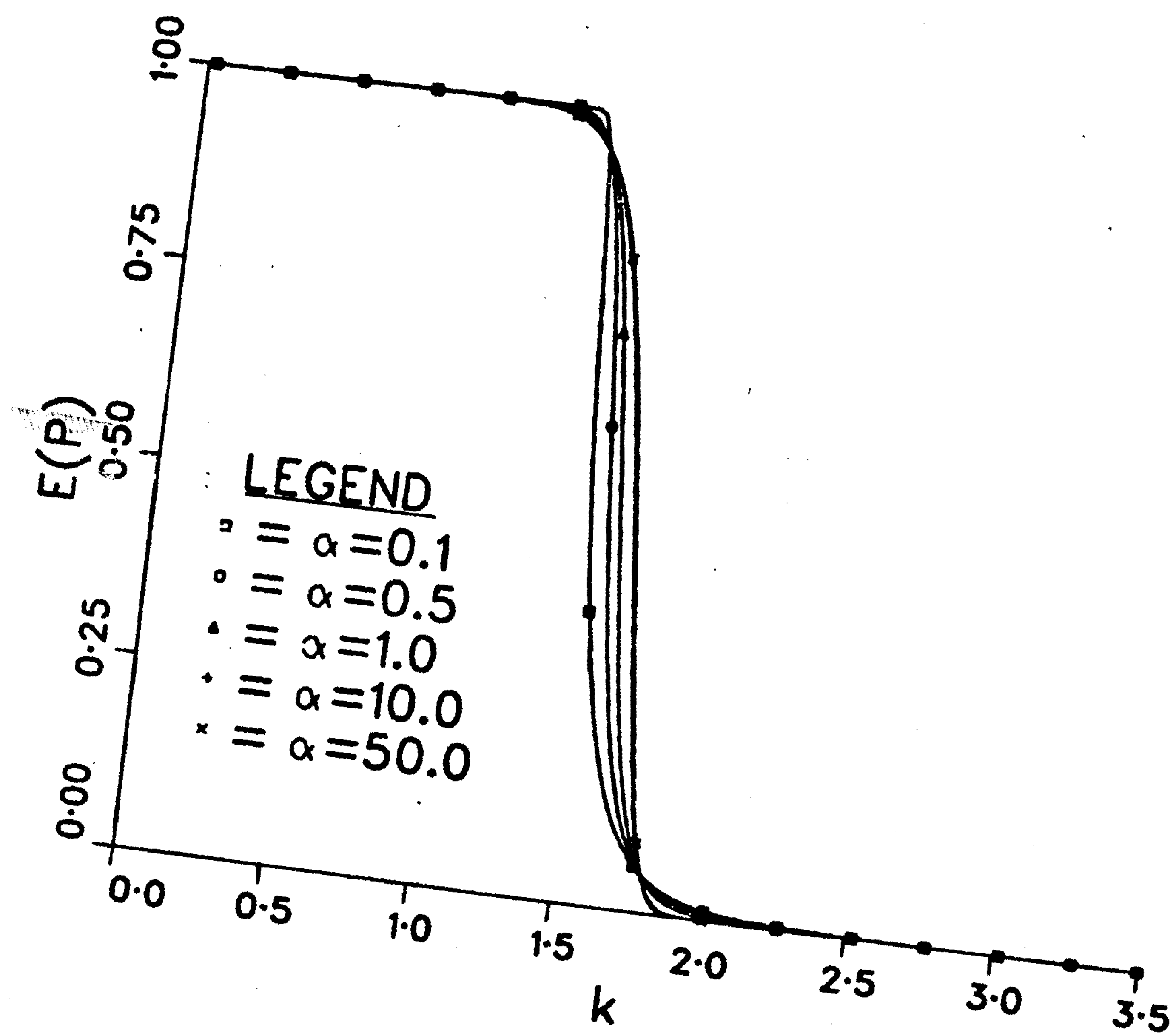
1·00
0·75
0·50
0·25
0·00
E(P)
LEGEND
= α=0.1
= α=0.5
= α=1.0
= α=10.0
= α=50.0
0·0
0·5
1·0
1·5
2·0
2·5
3·0
3·5
k

FIGURE 1

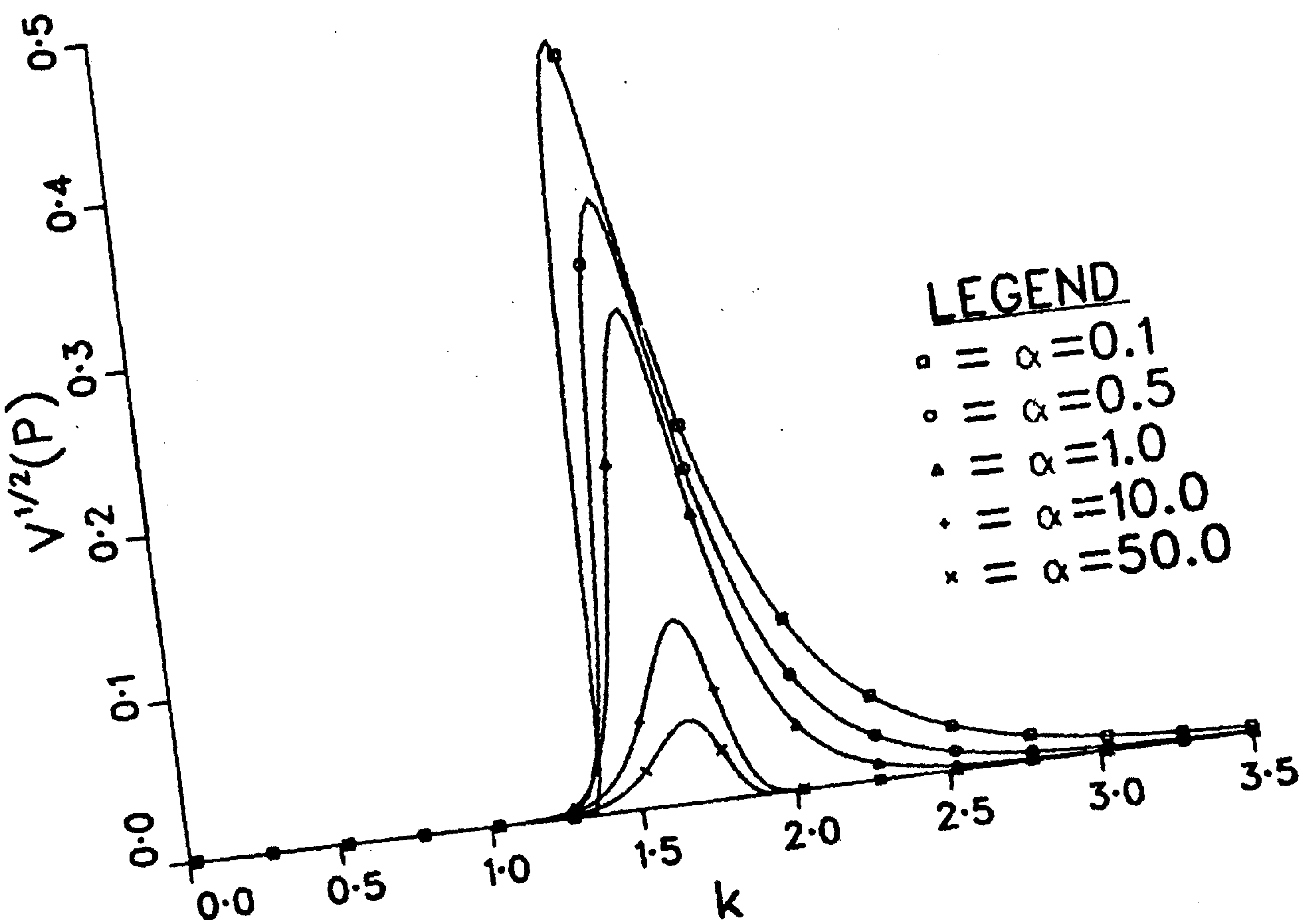
0·5
0·4
0·3
0·2
0·1
0·0
$V^{1/2}(P)$
0·0
0·5
1·0
1·5
2·0
2·5
3·0
3·5
k
LEGEND
= α=0.1
= α=0.5
= α=1.0
= α=10.0
= α=50.0

FIGURE 2

The model for use in obtaining the required test stress level is based on the following two propositions:

- If the test unit survives a test stress which exceeds the design stress, then remaining units should have a higher expected probability of survival at the design stress than if the test unit had been tested and survived at the design stress.
- If the test unit does not survive a test stress which exceeds the design stress, then remaining units should have either the same or a higher expected probability of survival at the design stress than if the test unit had been tested and failed at the design stress.

In both propositions, the increase depends upon the difference between the test and design stress levels. Mathematically, let us quantify the first proposition above according to

$$E(P_1|S_k) = g_1(k)E(P_1|S_1), k \geq 1, \tag{12}$$

where $g_1(k)$ is a suitably chosen function of k which has the following properties: (i) $g_1(1) = 1$; (ii) $g_1(k) \to 1/E(P_1|S_1)$ as $k \to \infty$; and (iii) $g_1'(k) \geq 0$, at all points of continuity of $g_1(k)$. The second property guarantees that, if the test unit survives a test of infinite stress, then the remaining units are expected to survive the design stress with probability equal to 1.

Similarly, the second proposition may be quantified as

$$E(P_1|\tilde{S}_k) = g_2(k)E(P_1|\tilde{S}_1), k > 1, \tag{13}$$

where $g_2(k)$ is a suitably chosen function of k which satisfies the following properties: (i) $g_2(1) = 1$; (ii) $g_2(k) \to E(P_1)/E(P_1|\tilde{S}_1)$ as $k \to \infty$; and (iii) $g_2'(k) \geq 0$, at all points of continuity of $g_2(k)$. The second property insures that, if the test unit fails a test of infinite stress, then nothing additional has been learned from the test about the ability of remaining units to survive the design stress. Particular choices for $g_1(k)$ and $g_2(k)$ will be considered in Secs. III and IV.

Now let us consider the optimization model itself. We wish to determine the value of k which maximizes the expected probability that S_1 occurs, given that we test a single unit at test stress $k.s_o$. That is, we wish to maximize

$$E(P_1;\alpha,\beta,\delta) = E(P_1|S_k;\alpha,\beta,\delta)\text{Prob}(S_k) + E(P_1|\tilde{S}_k;\alpha,\beta,\delta)\text{Prob}(\tilde{S}_k). \quad (14)$$

Upon substituting (8)-(13) into this expression and simplifying, we obtain

$$E(P_1;\alpha,\beta,\delta) = [\gamma_1 g_1(k)-\gamma_2 g_2(k)](1+\beta k^{\delta})^{-\alpha} + \gamma_2 g_2(k), \quad (15)$$

where

$$\gamma_1 = \left(\frac{1+\beta}{1+2\beta}\right)^{\alpha}, \quad (16)$$

and

$$\gamma_2 = \frac{1-\left(\frac{1+\beta}{1+2\beta}\right)^{\alpha}}{(1+\beta)^{\alpha}-1} = \frac{1-\gamma_1}{(1+\beta)^{\alpha}-1}. \quad (17)$$

In order to maximize $E(P_1;\alpha,\beta,\delta)$, it is useful to solve

$$\frac{\partial E(P_1;\ \alpha,\ \beta,\ \delta)}{\partial k} = [\gamma_1 g_1'(k)-\gamma_2 g_2'(k)](1+\beta k^{\delta})^{-\alpha}$$

$$-\alpha\beta\delta[\gamma_1 g_1(k) - \gamma_2 g_2(k)]\frac{k^{\delta-1}}{(1+\beta k^{\delta})^{\alpha+1}} + \gamma_2 g_2'(k) = 0. \quad (18)$$

The solution to (18) yields the desired optimal test stress k_o. The solution to the problem of finding k such that (14) is maximized is discussed in the next section.

III. FITTING THE MODEL

First, let us consider a procedure for estimating α, β, and δ. Since only a single test unit is assumed to be available, the suggested procedure is necessarily subjective. Consider the following two questions:

<u>Question 1</u>: Prior to the test, at what stress level k_1 will the test unit have approximately a 95% expected chance of survival?

<u>Question 2</u>: Prior to the test, at what stress level k_2 will the test unit have approximately a 5% expected chance of survival?

Now $k_2 > k_1$ and both are expressed in units of design stress. Then

$$(1+\beta\, k_i^{\delta})^{-\alpha} = \xi_i,\ i=1,2,$$

where $\xi_1 = .95$ and $\xi_2 = .05$. Then

$$\beta = (\xi_i^{-1/\alpha}-1)k_i^{-\alpha} \quad i=1,2, \tag{18}$$

and further simple algebra yields

$$\delta = \frac{\ln[(\xi_1^{-1/\alpha}-1)/(\xi_2^{-1/\alpha}-1)]}{\ln[k_1/k_2]} \tag{19}$$

Therefore we have $\beta=\beta(\alpha)$ and $\delta=\delta(\alpha)$, so that our three parameter model has been reduced to a one parameter model.

Since the prior variance of the survival probability is given by

$$V(p;\alpha,\beta,\delta), = V(\alpha) = (1+2\beta k^{\delta})^{-\alpha} - (1+\beta k^{\delta})^{-2\alpha},$$

the parameter α may be chosen to coincide with the experimenter's prior estimate of the variation at some stress k. For the examples we have worked out, $V(\alpha)$ has been observed to be a decreasing function of α.

Two other functions, $g_1(k)$ and $g_2(k)$, must be specified. In our calculations, we have taken

$$g_1(k) = k^c, \qquad k \leq K$$

and

$$g_2(k) = 1, \qquad k \leq K$$

where K is an unspecified upper limit of test stress beyond the range of practical interest. For $k>K$, suitable adjustments would have to be made to these choices of $g_1(k)$ and $g_2(k)$ to ensure that the appropriate asymptotic properties discussed in Section II are present.

Recall that

$$E(P_1|S_k) = g_1(k)\ E(P_1|S_1)$$
$$= k^c\ E(P_1|S_1)$$

The choice $g_2(k) = 1$ expresses the situation in which, if the test unit fails a test of stress $k, 1<k\leq K$, then the expected probability of survival is the same as if the test unit had failed a test at the design stress.

To determine c, consider the following question:

Question 3: Prior to the test, suppose that a hypothetical test unit was tested and survived an increased test stress level. At what stress level k_3 will the expected failure probability, $E(1-P_1|S_{k_3})$, be one-half as large as the expected failure probability of a hypothetical unit which was tested and survived the design stress, $E(1-P_1|S_1)$?

Of course, k_3 must also be expressed in units of design stress. Then

$$1 - E(P|S_1) = 2\left\{1 - E(P|S_{k_3})\right\}$$

or

$$1 - \left(\frac{1+2\beta}{1+\beta}\right)^{-\alpha} = 2\left\{1 - k_3^c\left(\frac{1+2\beta}{1+\beta}\right)^{-\alpha}\right\} .$$

Some elementary algebra yields

$$c = \ln\left[\frac{\left(\frac{1+2\beta}{1+\beta}\right)^{\alpha} + 1}{2}\right] / \ln[k_3] \ . \tag{20}$$

Therefore, for the specific choice of $g_1(k)$ and $g_2(k)$, the parameters β, δ, and c are given by equations (18), (19), and (20). Then, when α is chosen in accord with the experimenter's estimate of the variation, the parameters of the model are completely determined.

IV. EXAMPLE

As indicated in the introduction, the example considered here concerns the fuel container of a radio-isotope thermoelectric generator (RTG) system used as the power supply for a space satellite. In the radioisotope fuel container there are a number of fuel elements, which are simply spheres which contain the fuel. The RTG is designed to withstand impact onto flat plate steel at a certain velocity so that, for example, launch pad accidents will not release radioactive material to the environment.

The answers to questions 1, 2, and 3 of Section III were solicited from a group of engineers at the Los Alamos Scientific Laboratory responsible for such impact tests. The answers for one particular RTG system of interest were as follows:

1) At approximately 140 fps, the test unit should have roughly a 95% expected chance of survival.

2) At approximately 180 fps, the test unit should have roughly a 5% expected chance of survival.

3) At approximately 135 fps, $E(1-P_1|S_{k_3})$ should be roughly one-half of $E(1-P_1|S_1)$.

The design stress velocity S_o is 100 fps. Thus, k_1=1.4, k_2=1.8, and k_3=1.35. Equation (14), for g_1 and g_2 specified in section III, becomes

$$E\left(P_1;\ \alpha,\beta(\alpha),\delta(\alpha)\right) = k^c\left(\frac{1+\beta}{1+2\beta}\right)^{\alpha}(1+\beta k^{\delta})^{-\alpha} + \left(1-\left(\frac{1+\beta}{1+2\beta}\right)^{\alpha}\right)\left((1+\beta)^{\alpha}-1\right)^{-1}\left(1-(1+\beta k^{\delta})^{-\alpha}\right) \tag{21}$$

The optimal k, k_o, is found on a computer by a simple search program.

Certain α and k_1,k_2, k_3 result in difficulties in computation. For example, if βk^{δ}=0, then on the computer

$$1 - (1+\beta k^{\delta})^{-\alpha} = 0$$

whereas a simple expansion shows that

$$1 - (1+\beta k^{\delta})^{-\alpha} \simeq + \alpha\beta k^{\delta}$$

While such approximations were used whenever possible, it was not possible to obtain all values in the tables below. A "*" indicates that the computation was not performed.

Table 1 gives the values of $\beta=\beta(\alpha)$, $\delta=\delta(\alpha)$, $c=c(\alpha)$, and $k_o=k_o(\alpha)$ for several different choices of α. The values of k_i are k_1=.14, k_2=1.8, and k_3=1.35. Observe that k_o is a very stable function of α.

TABLE 1

VALUES OF β, δ, c, AND OPTIMAL STRESS k_o FOR SELECTED VALUES OF α

α	β	δ	c	k_o
.3	3.07×10^{-8}	46.42	1.53×10^{-8}	1.96
.4	4.22×10^{-7}	37.71	2.81×10^{-7}	1.35
.5	.0000018	32.69	.0000015	1.36
.6	.0000044	29.45	.0000044	1.37
.7	.0000080	27.23	.0000093	1.38
.8	.0000120	25.61	.0000159	1.38
.9	.0000160	24.38	.0000240	1.39
1.0	.0000198	23.43	.0000330	1.39
2.0	.0000370	19.48	.0000123	1.43
3.0	.0000365	18.30	.0001825	1.45
4.0	.0000330	17.74	.0002198	1.46
5.0	.0000294	17.42	.0002450	1.47
10.0	.0000181	16.79	.0003021	1.48
15.0	.0000129	16.58	.0003232	1.49
20.0	.0000100	16.48	.0003342	1.49
25.0	.0000082	16.42	.0003409	1.49
30.0	.0000069	16.38	.0003454	1.49
35.0	.0000060	16.35	.0003487	1.49
40.0	.0000053	16.33	.0003512	1.49
45.0	.0000047	16.32	.0003530	1.49
50.0	.0000042	16.30	.0003546	1.49

Values of the prior standard deviation, $V^{1/2}(\alpha)$, are plotted as a function of stress k in Figure 2 for several choices of α. As can be seen, the prior standard deviation decreases as α increases. Figure 3 gives a plot of $E(P)$, $E(P|S_k)$ and $E(P|\tilde{S}_k)$ as a function of stress k for α=0.1. It is observed that

$$E(P|\tilde{S}) \leq E(P) \leq E(P|S).$$

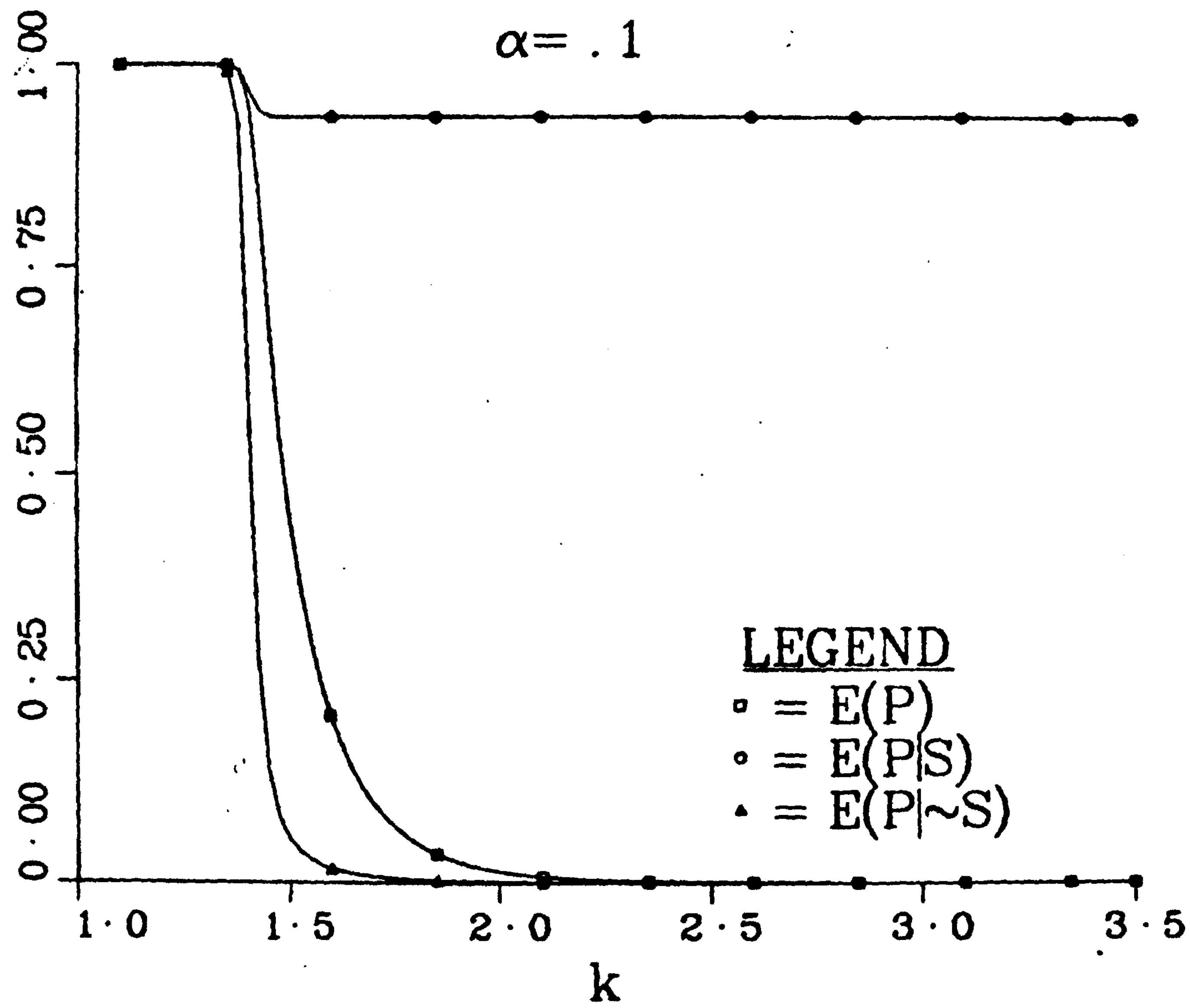

FIGURE 3

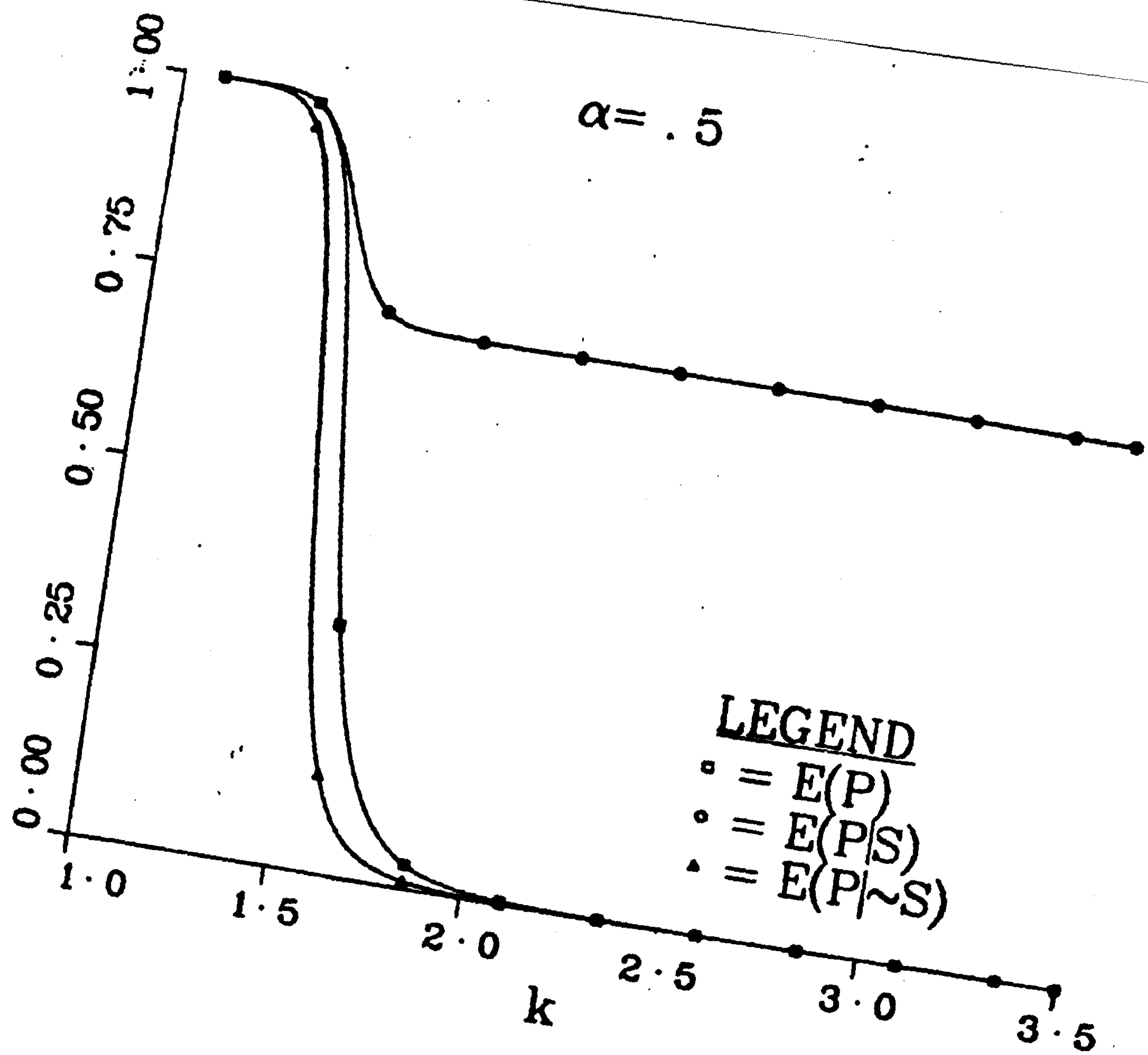

FIGURE 4

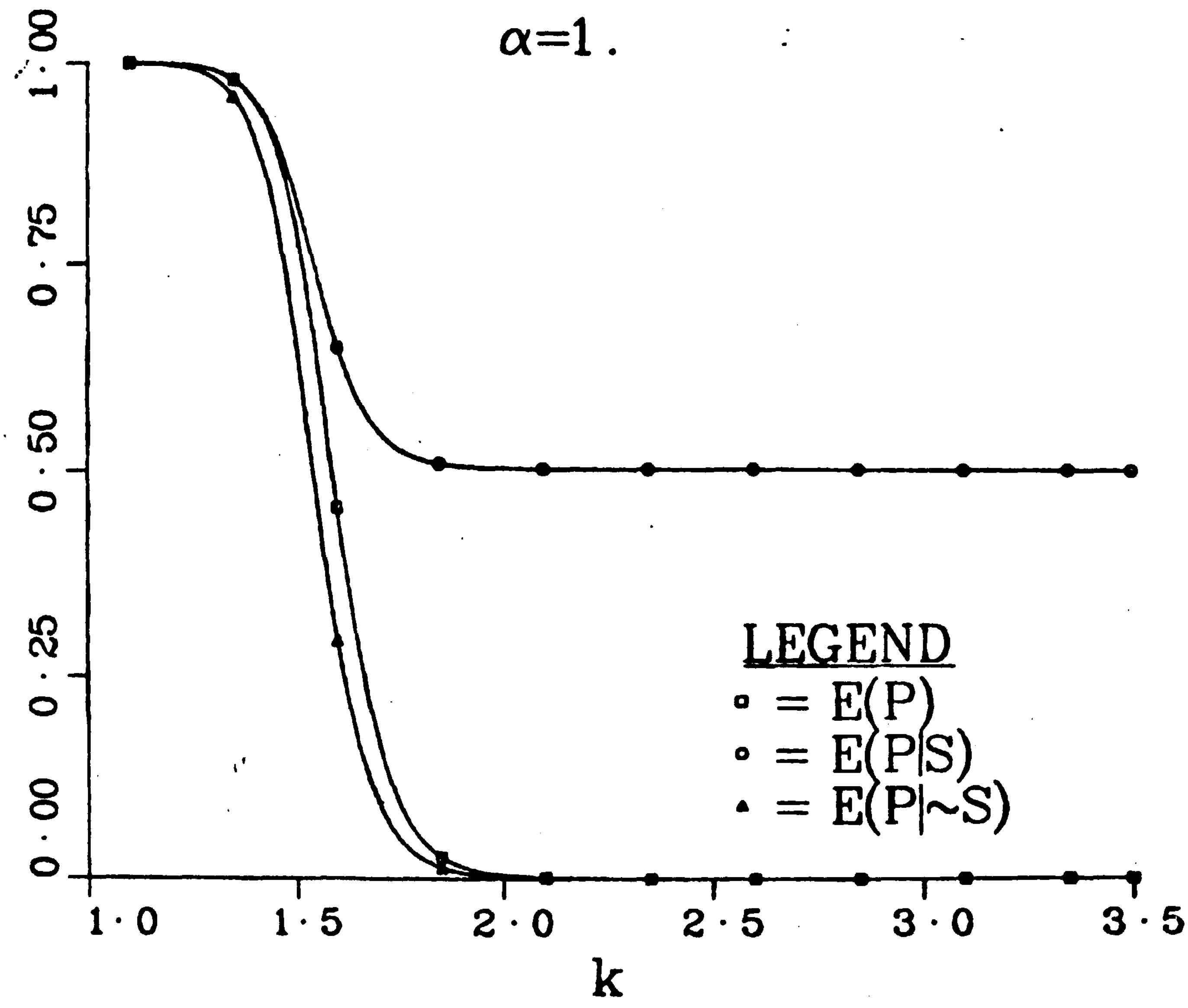

FIGURE 5

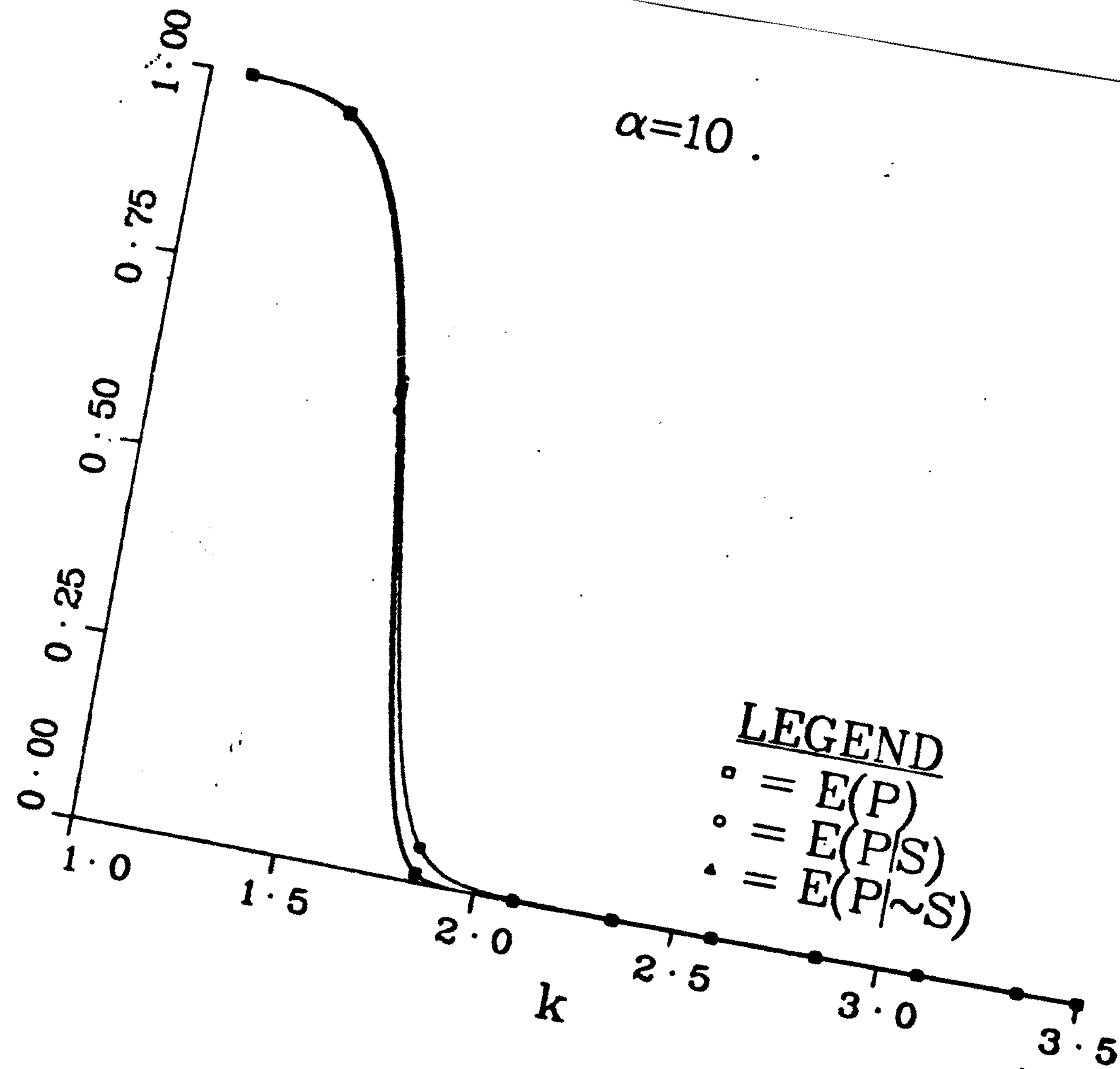

FIGURE 6

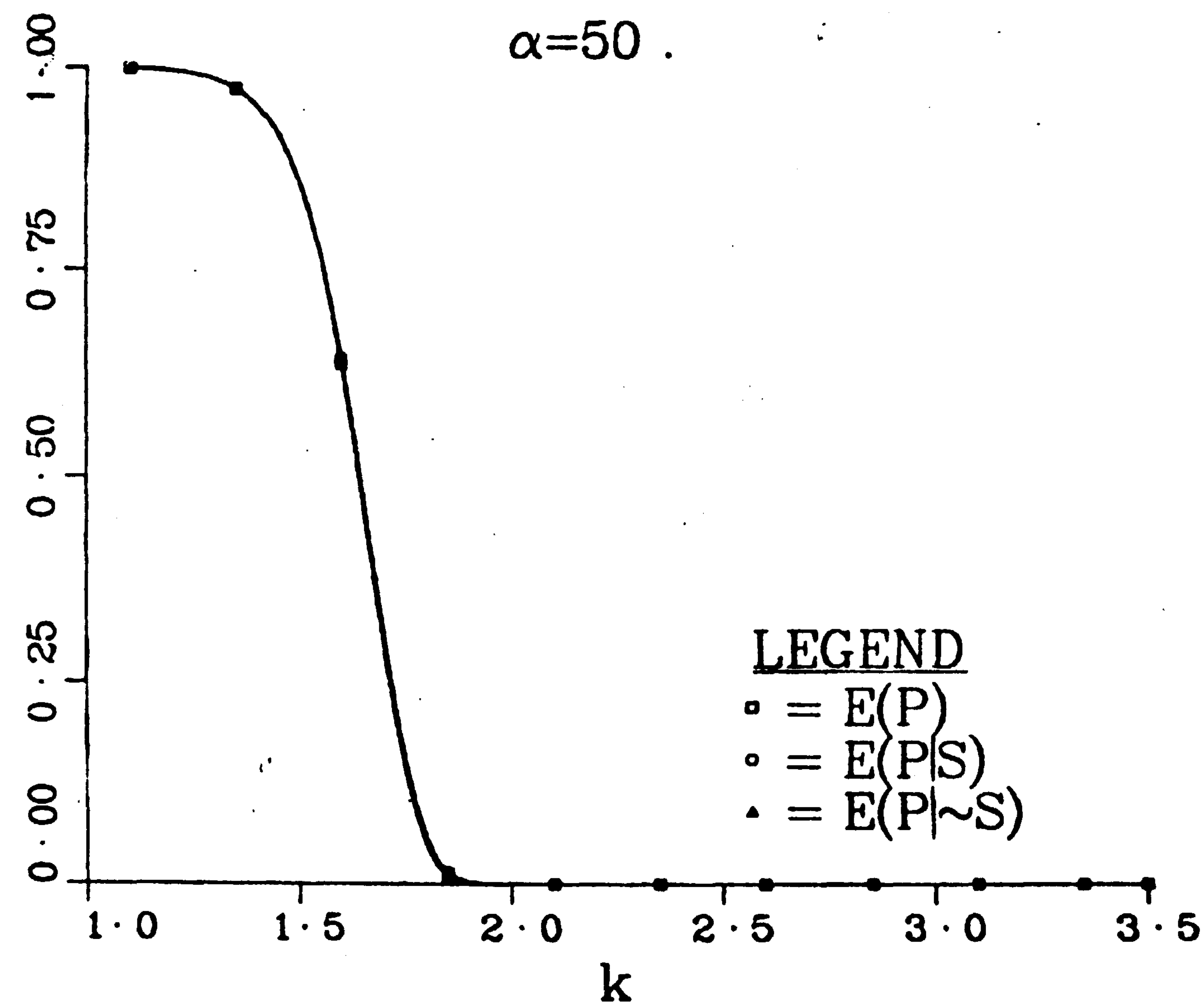

FIGURE 7

Also, it is easy to see that the asymptote for E(P|S) is

$$\lim_{k\to\infty} E(P|S) = \lim_{k\to\infty} \left(\frac{1+2\beta k^{\delta}}{1+\beta k^{\delta}}\right)^{-\alpha} = 2^{-\alpha}.$$

Figures 4-7 give plots similar to Figure 3 for α = 0.5, 1.0, 10.0. and 50.0, respectively.

Earlier, in section III, we suggested the α could be chosen to coincide with the experimenter's prior estimate of the variance at a given stress k. It is also possible to choose α by use of Figures 3-7. From these figures, it is observed that the difference between the prior and posterior expected survival probabilities is larger for smaller values of α. That is, for small values of α, the expected survival probability is more sensitive to the test result than for large values of α.

Let us now examine the sensitivity of the optimal test stress k_0, as given by the solution to (21), to variations in the answers to questions 1, 2, and 3. It is important to do this since the answers to these questions may be inaccurate. Such inaccuracy may be due to either lack of precise knowledge by the person(s) answering the questions or lack of clear understanding of the precise information being solicited in the questions.

First, consider the sensitivity of the optimal test stress to changes in k_1 and/or k_2. Tables 2-5 give the optimal test stress k_0 as a function of several choices of k_1 and k_2 for α = 0.5, 1.0, 10.0, and 50.0, respectively. In Tables 2-5, k_3 = 1.35.

TABLE 2.

OPTIMAL TEST STRESS k_0 FOR SEVERAL VALUES OF k_1 AND k_2 (α = 0.5 AND k_3 = 1.35)

		k_1				
		1.2	1.3	1.4	1.5	1.6
	1.6	1.17	1.26	1.87	*	*
	1.7	1.18	1.27	1.36	*	*
k_2	1.8	1.18	1.27	1.36	*	*
	1.9	1.19	1.27	1.37	*	1.90
	2.0	1.19	1.28	*	1.46	1.93

TABLE 3.

OPTIMAL TEST STRESS k_0 FOR SEVERAL VALUES OF k_1 AND k_2 (α = 1.0 AND k_3 = 1.35)

		k_1				
		1.2	1.3	1.4	1.5	1.6
	1.6	1.21	1.29	*	*	*
	1.7	1.23	1.30	1.38	1.96	*
k_2	1.8	1.25	1.32	1.39	1.51	1.81
	1.9	1.26	1.33	1.41	1.49	1.63
	2.0	1.28	1.35	1.42	1.50	1.58

TABLE 4.

OPTIMAL TEST STRESS k_0 FOR SEVERAL VALUES OF k_1 AND k_2 (α = 10.0 AND k_3 = 1.35)

		k_1				
		1.2	1.3	1.4	1.5	1.6
	1.6	1.32	1.36	1.42	1.21	*
	1.7	1.36	1.40	1.45	1.81	1.08
k_2	1.8	1.40	1.44	1.48	1.54	1.84
	1.9	1.44	1.47	1.52	1.57	1.62
	2.0	1.47	1.51	1.55	1.60	1.65

TABLE 5.

OPTIMAL TEST STRESS k_0 FOR SEVERAL VALUES OF k_1 AND k_2 (α = 50.0 AND k_3 = 1.35)

		k_1				
		1.2	1.3	1.4	1.5	1.6
	1.6	1.34	1.37	1.43	*	*
	1.7	1.38	1.41	1.46	*	1.09
k_2	1.8	1.42	1.45	1.49	1.54	1.23
	1.9	1.46	1.49	1.53	1.58	1.63
	2.0	1.50	1.53	1.57	1.61	1.66

It is observed that the optimal test stress ranges between 1.09 (α = 50, k_1 = 1.6, k_2 = 1.7) and 1.93 (α = 0.5, k_1 = 1.6, k_2 = 2.0). For a given α, the optimal test stress is fairly insensitive to changes in k_2 for small values of k_1. On the other hand, for a given α, the optimal test stress is fairly insensitive to changes in k_1 for large values of k_2. As both k_1 and k_2 increase, the optimal test stress is fairly stable.

Now consider the sensitivity of the optimal test plan to changes in k_3, since this quantity was held fixed in Tables 2-5. Table 6 gives the optimal test stress k_0 as a function of several choices of k_3 and α for the nominal values k_1 = 1.4 and k_2 = 1.8. It is observed that k_0 is quite insensitive to

TABLE 6.

OPTIMAL TEST STRESS k_0 FOR SEVERAL VALUES OF k_3 AND α (k_1 = 1.4 AND k_2 = 1.8)

		k_3					
		1.10	1.20	1.30	1.35	1.40	1.50
	0.5	1.39	1.38	1.36	1.36	1.36	1.35
α	1.0	1.43	1.41	1.40	1.39	1.39	1.38
	10.0	1.49	1.49	1.48	1.48	1.48	1.48
	50.0	1.50	1.50	1.49	1.49	1.49	1.49

changes in k_3 for a fixed value of α. This is an important result, since the answer to Question 3 is likely to be somewhat arbitrary in practice. That is, in practice, k_3 may be an imprecisely known value.

V. CONCLUSIONS

A Bayesian procedure for determining the optimal test stress for a single test unit has been developed. The procedure is both objective as well as subjective. It is subjective in the sense that the necessary parameters in the model are estimated from best available information prior to the test results. These estimates are then used in an objective manner to provide the required test stress. The test stress provided by this procedure is "optimal" within the model framework in a certain well-defined sense. Namely, this optimal test stress maximizes the modeled expected unconditional probability of survival at the design stress. The model effectively trades between two extremes. The first represents the increasing likelihood of survival at the design stress gained as a result of a test unit surviving increasing test stress. This gain is countered by the correspondingly decreasing probability of test unit survival as the test stress increases.

The model was used to determine the impact test velocity in an impact test of a certain radioisotope fuel container. The optimal test velocity was found to be approximately 30-50 percent above the design impact velocity. In addition, a limited sensitivity analysis to the subjective estimates required in fitting the model was conducted. It was observed in this example that the optimal test velocity was fairly insensitive to the subjective estimates. This may or may not be true in other applications. Consequently, as a safeguard, it is recommended that such a sensitivity analysis be routinely conducted when applying this model in practice.

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Consider the case of a single test unit which must be tested at some level of test stress. Suppose that the test stress level is free to be determined, and that only the survival or nonsurvival of the unit is observed. It is assumed that the unit is designed to withstand a known and specified design stress level. A Bayesian model is developed for determining the required level of test stress which maximizes the expected probability of survival at the design stress level. Engineering experience

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20. ABSTRACT (Continued)

from similar past tests on similar units is used to fit the model. A practical application illustrates the method. The sensitivity of the procedure to changes in the parameters used in fitting the model is also examined. The procedure is fairly insensitive to three parameters required in fitting the model in the example.